A Mathematical Model for the Control of the Spread of Meningitis Virus Disease in West Africa- A Disease Free Equilibrium and Local Stability Analysis Approach

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Abstract: In this research work, we developed and analysed a deterministic model for controlling the spread of meningitis virus disease in a population using disease free equilibrium and local stability analysis approach. The entire population was partitioned into six compartments which are: susceptible (S), vaccinated individuals (V), Carriers (C), infected individuals (I), treated individuals (T) and recovered individual (R) and every newborn in each compartment is previously infection free by assumption. A mathematical analysis of disease free equilibrium, basic reproduction number and local stability analysis were carried out. The results show that the disease free equilibrium state is stable and the endemic equilibrium state is locally asymptotically stable.

Keywords*: Disease free equilibrium, Basic reproduction number, stability analysis, Jacobian matrix, meningitis, vaccination, carrier, treatment, equilibrium state.*

Introduction

Meningitis is a killer disease that affects the delicate membranes of children and adults called meninges which cover the brain and spinalchord[1]. Meningitis is always caused by a bacterial or virious that develops within the body close to the brain like throat, sinuses or nose or ears. Other causes of meningitis are fungal infection, syphilis, tuberculosis autoinniene disorder, cancer medication.

Meningitis is classified into into different types which are bacterial meningitis, viral meningitis, parasitic meningitis, fungal meningitis and non-infectious meningitis[11]

Bacterial meningistis:It is a dangerous life threatening disease which can lead to brain damage under poor treatment. In a developed country like US, itis caused by strephcoccus pneumonia(pneumococcus), Neisseria meningitides (meningococcus) and cisteriamonocytogenes(which majorly affects pregnant women, old people or those suffering immune system problems) [3]

Bacterial menigistics begins when the bacterial infect the blood stream from the sinuses, ears or throat. The bacteria then goes through the blood stream to the brain and reside. It spreads when somebody who is infected cough or sneeze.

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Meningitis is dangerious for people with certain health conditions such as a long-term disease, damaged or missing spleen or people with immune system disorder. Since the germs that cause meningitis easily spread, out breaks are most likely to occur in a clustered settlement such as military recruits in a barracks, university students in hostels etc.[2]

Menigitis transmission passes through some steps. In step one an individual has to be susceptible to meningitis disease. The three main bacteria mentioned earlier which causes meningitis may be present in the environment if a susceptible individual comes into contact with any of the bacterial he or she becomes a carrier[7]

Beginning from September 1, 2017 to January 25, 2018 at total of 484 suspected cases have been reported from twelve states;Zamfara (257), Kastina (86), Sokoto(41), Jigawa (29), Bauchi (20), Cross River (17), Kebbi(12), Yobe(9), Kano (7), Borno (3), Adamawa (2) and Kaduna (1), out of the 481 suspected cases, 176 (36.6%) samples have been collected and tested of the 176 samples tested, 79 (44.9%) were positive for bacterial meningitis the subject matter. Though reactive vaccination has been done in those areas outbreak summary reviewed that between September, 2017 (Epi-wk 36) and January, 2018 (Epi-wk 3) a total of 481d cases with 82 death (CFR=17.0%) have been reported. The most affected group is 1-15 years (74.0%) with male to female ratio of 1.5:1.0, currently no LGA is in epidemic threshold due to control measure being implemented in the affected states. [8]. The aim of this research work is to: contribute the global effort in controlling the spread of the killer disease, meningitis virus disease in Nigeria in particular and in the whole world in general using a mathematical model.

Model Formulation

In this work, the total population (N) is categorized into six compartments susceptible individual (S) vaccinated humans(V) carrier individual (C) infected individual (I), Treated individual (T), Recovered individual (R). Therefore

$$N(t) = S(t) + V(t) + C(t) + I(t) + T(t) + R(t)$$

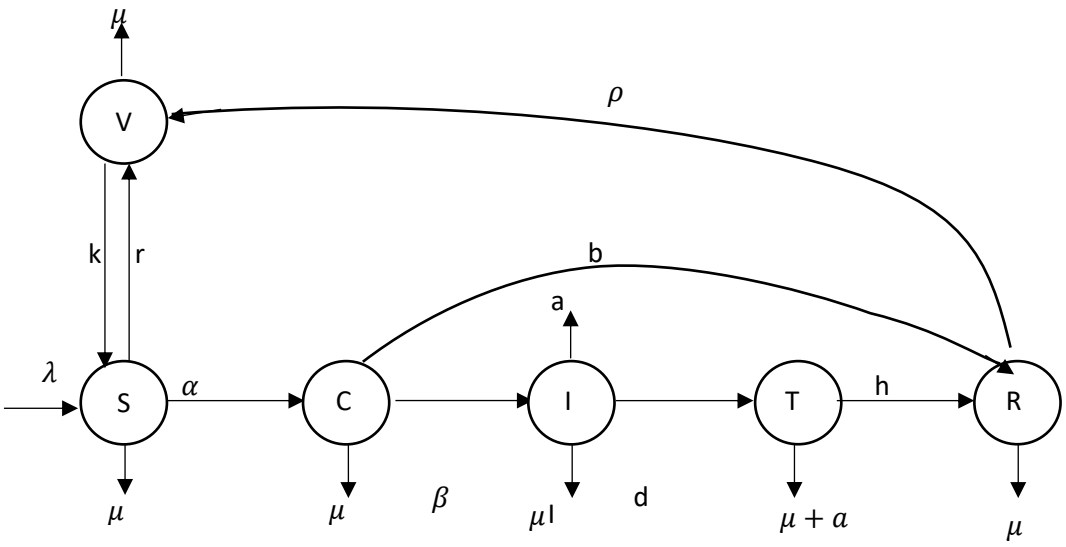

Fig I. Schematic representation of the model

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Variables and Parameters

The variables and parameters used in this model with descriptions are:

Parameter	Description
$S(t)$:	The population of susceptible individuals at time t.
$V(t)$:	The population of vaccinated individuals at time t.
$C(t)$:	The population of carriers individuals at time t.
$I(t)$:	The population of infected individuals at time t.
$T(t)$:	The population of treated individuals at time t.
$R(t)$:	The population of individuals who have been recovered from infection after treatment at time t.
r:	The rate at which susceptible individuals become vaccinated
k:	The rate at which vaccinated individuals become susceptible due to vaccine failure.
λ:	The recruitment rate: that is the population of newborn babies enteringthe entire population
μ:	The natural death rate
β:	The infection rate

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a: The disease induce death rate

d: The rate at which infected individuals become treated

b: The rate at which carriers become recovered

ρ: The rate at which recovered individuals become vaccinated

$N(t)$: The total population

ω; Probability of infectious individual infecting others

c: Per capital contact rate

α The rate at which the susceptible become carriers

N The total population

H Rate at which treated individuals recovered

Model Assumptions

i. The population mixture is homogenous.

ii. That the individuals in a compartment have equal natural death rate.

iii. That there is presently vaccine that provides immunity against meningitis virus disease.

iv. That all newborns are previously infection free and therefore can only be recruited into the susceptible population.

v. The susceptible individuals that are vaccinated can also be susceptible due to vaccine failure.

vi. Sex, age, race and social status do not affect the probability of any one being infected by meningitis virus.

vii. That treatment existsforinfected individuals and the disease can be cured.

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viii. That there is no emigration or immigration in the population

Base on the model assumptions and the interactions of each compartment on the schematic diagram above, we can therefore formulate the following differential equation for meningitis virus diseases dynamics.

$$\frac{dS}{dt} = \lambda + kV - (\alpha + r + \mu)S \quad (1)$$

$$\frac{dV}{dt} = rS + \rho R - (k + \mu)V \quad (2)$$

$$\frac{dC}{dt} = \alpha S - (\beta + b + \mu)C \quad (3)$$

$$\frac{dI}{dt} = \beta C - (a + d + \mu)I \quad (4)$$

$$\frac{dT}{dt} = dI - (\mu + h + a)T \quad (5)$$

$$\frac{dR}{dt} = bC + hT - (\mu + \rho)R \quad (6)$$

Where $\alpha = \frac{\omega c\,(C+I+T)}{N}$

Equilibrium Point

At equilibrium state we have

$$\frac{ds}{dt} = \frac{dV}{dt} = \frac{dC}{dt} = \frac{dI}{dt} = \frac{dT}{dt} = \frac{dR}{dt} = 0$$

So that

$$\lambda + kV - (\alpha + r + \mu)S = 0 \quad (7)$$

$$rS + \rho R - (k + \mu)V = 0 \quad (8)$$

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$$\alpha S - (\beta + b + \mu)C = 0 \quad (9)$$

$$\beta C - (a + \rho + \mu)I = 0 \quad (10)$$

$$dI - (a + \mu + h)T = 0 \quad (11)$$

$$bC + aT - (\mu + \rho)R = 0 \quad (12)$$

Disease Free Equilibrium Point

The disease free equilibrium point is the point of total disease free denoted E^*

Therefore $E_0^* = (S^*, V^*, C^*, I^*, T^*, R^*)$

At disease free state

$$C^* = I^* = T^* = R^* = 0 \quad (13)$$

Solving equation (7) - (8) we have

$$E_0 = \left[\frac{\lambda(k+\mu)}{(k+\mu)(\alpha+r+\mu)-kr}, \frac{r\lambda}{(\alpha+r+\mu)-kr}, 0,0,0,0\right] \quad (14)$$

EXISTENCE OF ENDEMIC EQUILIBRIUM POINT (EEP)

From equations 7 to 12 we have

$$S^* = \frac{N(\beta+b+\mu)(a+d+\mu)(a+\mu+h)}{\omega c(a+\mu+h)(a+d+\mu)+\omega c\beta(a+\mu+h)+\omega c\beta d}$$

$$V^* = \frac{(\alpha+r+\mu)[N(\beta+b+\mu)(a+d+\mu)(a+\mu+h)]}{k\omega c(a+\mu+h)(a+d+\mu)+ k\omega c\beta(a+\mu+h)+ k\omega c\beta d}$$

$$C^* = \frac{\alpha N(\beta+b+\mu)(a+d+\mu)(a+\mu+h)}{(\beta+b+\mu)[\omega c(a+\mu+h)(a+d+\mu)+\omega c\beta(a+\mu+h)+\omega c\beta d]}$$

$$I^* = \frac{\alpha\beta N(\beta+b+\mu)(a+d+\mu)(a+\mu+h)}{(\beta+b+\mu)(a+d+\mu)[\omega c(a+\mu+h)(a+d+\mu)+\omega c\beta(a+\mu+h)+\omega c\beta d]}$$

$$T^* = \frac{\alpha\beta dN(\beta+b+\mu)(a+d+\mu)(a+\mu+h)}{(\beta+b+\mu)(a+d+\mu)(a+h+\mu)[\omega c(a+\mu+h)(a+d+\mu)+\omega c\beta(a+\mu+h)+\omega c\beta d]}$$

$$R^* = \frac{(k+\mu)(\alpha+r+\mu)[N(\beta+b+\mu)(a+d+\mu)(a+\mu+h)]}{\rho[k\omega c(a+\mu+h)(a+d+\mu)+ k\omega c\beta(a+\mu+h)+ k\omega c\beta d]} - \frac{rN(\beta+b+\mu)(a+d+\mu)(a+\mu+h)}{\rho[\omega c(a+\mu+h)(a+d+\mu)+\omega c\beta(a+\mu+h)+\omega c\beta d]}$$

The basic reproduction number R_0

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The basic reproduction number the average number of secondary infections produced when one infected individual is introduced into uninfected population.

If $R_0 < 1$ the E_0 is stable, if $R_0 > 1$ then E_0 is unstable [5]

We calculate R_0 using the next generation matrix method defined by the spectral radius of FV^{-1} such that

$$f_i = \begin{pmatrix} \frac{\omega c}{N}(C + I + T)S_0 \\ \beta c \end{pmatrix}$$

$$V_i = \begin{pmatrix} (\beta + b + \mu)C \\ (a + d + \mu)I \end{pmatrix}$$

Differentiating f_i and V_i w r t C and I we have

$$F = \begin{pmatrix} \frac{\omega c}{N}S_0 & \frac{\omega c}{N}S_0 \\ \beta & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} (\beta + b + \mu) & 0 \\ 0 & (a + d + \mu) \end{pmatrix}$$

So that

$$FV^{-1} = \begin{pmatrix} \frac{\omega c}{N(\beta + b + \mu)}S_0 & \frac{\omega c}{N(a + d + \mu)} \\ \frac{\beta}{(\beta + b + \mu)} & 0 \end{pmatrix}$$

R_0 is the spectra radius of $|FV^{-1} - I\lambda| = 0$

$$R_0 = \frac{1}{2}\left(\frac{\omega c}{N(\beta + b + \mu)} + \sqrt{\left(\frac{\omega c}{N(\beta + b + \mu)}\right)^2 - \frac{4\beta\omega c}{N(\beta + b + \mu)(a + d + \mu)}S_0} \right)$$

Local Stability Analysis of the Disease Free Equilibrium State

We find the Jacobian matrix of the model equations in other to determine the local stability analysis

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$$J(\varepsilon F) = \begin{bmatrix} -(\alpha + r + \mu) & k & 0 & 0 & 0 & 0 \\ r & -(k+\mu) & 0 & 0 & 0 & \rho \\ \alpha & 0 & -(\beta + b + \mu) & 0 & 0 & 0 \\ 0 & 0 & \beta & -(a + d + \mu) & 0 & 0 \\ 0 & 0 & 0 & d & -(\mu + h) & 0 \\ 0 & 0 & b & 0 & d & -(\mu + \rho) \end{bmatrix}$$

The characteristics equation is given by

$$J(\varepsilon F - \lambda I) = \begin{bmatrix} -(\alpha + r + \mu) & k & 0 & 0 & 0 & 0 \\ r & -(k+\mu) & 0 & 0 & 0 & \rho \\ \alpha & 0 & -(\beta + b + \mu) & 0 & 0 & 0 \\ 0 & 0 & \beta & -(a + d + \mu) & 0 & 0 \\ 0 & 0 & 0 & d & -(\mu + h) & 0 \\ 0 & 0 & b & 0 & d & -(\mu + \rho) \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} -(\alpha + r + \mu) - \lambda & k & 0 & 0 & 0 & 0 \\ r & -(k+\mu) - \lambda & 0 & 0 & 0 & \rho \\ \alpha & 0 & -(\beta + b + \mu) - \lambda & 0 & 0 & 0 \\ 0 & 0 & \beta & -(a + q + \mu) - \lambda & 0 & 0 \\ 0 & 0 & 0 & d & -(\mu + h) - \lambda & 0 \\ 0 & 0 & b & 0 & d & -(\mu + \rho) - \lambda \end{bmatrix}$$

The determinant yields.

$$\{-(\alpha + r + \mu) - \lambda\}\{-(k+\mu) - \lambda\}\{-(\beta + b + \mu) - \lambda\}\{-(a + d + \mu) - \lambda\}\{-(\mu + h) - \lambda\{-(\mu + \rho) - \lambda\} = 0$$

$-(\alpha + r + \mu) - \lambda = 0\ or - (k + \mu - \lambda = 0\ or - (\beta + b + \mu) - \lambda = 0$

$-(a + d + \mu) - \lambda = 0\ or - (\mu + h) - \lambda = 0\ or - (\mu + \rho) - \lambda = 0$

$\lambda_1 = -(\alpha + r + \mu)\ or\ \lambda_2 = (k + \mu)$

$\lambda_3 = -(\beta + b + \mu)\ or\ \lambda_4 = -(a + d + \mu)\ or\ \lambda_5 = -(\mu + h)\ \ or\ \lambda_6 = -(\mu + \rho)$

Therefore, $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6 < 0$which implies that the disease free equilibrium is stable

Stability Analysis of Non-Zero Equilibrium State

An important criterion by Routh-Hurwitz gives the necessary and sufficient conditions for all roots of the characteristics equations (with real coefficients) to lie in the left half of the complex plane. In other words, all the roots of the polynomial are negative or have real roots if and only if the determinant of all Hurwitz matrices is positive.

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Theorem 1 (Routh-Hurwitz Conditions)

Let $J = \begin{pmatrix} f_x(x_*, y_*) & f_y(x_*, y_*) \\ g_x(x_*, y_*) & g_y(x_*, y_*) \end{pmatrix}$ be the Jacobian matrix of the non-linear system

$$\frac{dx}{dt} = f(x, y)$$

$$\frac{dy}{dt} = g(x, y)$$

Evaluated at the critical point (x_*, y_*), then the critical point (x_*, y_*);

1. Is locally asymptotically stable if trace (J)<0 and determinant>0
2. Is stable but not asymptotically stable if trace (J)=0 and determinant (J)>0
3. Is unstable if either, trace (J)>0 or determinant (J)<0

Jacobean matrix of the system of equations at endemic equilibrium state is:

$$J\left(x^*_1, x^*_2, x^*_3, x^*_4, x^*_5, x^*_6\right) =$$

$$= \begin{bmatrix} -(\frac{\omega c(C^* + I^* + T^*)}{N}\alpha + r + \mu) & \alpha & 0 & 0 & 0 & 0 \\ r & -(k+\mu) & 0 & 0 & 0 & \rho \\ \frac{\omega c(C^* + I^* + T^*)}{N}\alpha + r + \mu & r & -(\beta + b + \mu) & 0 & 0 & 0 \\ 0 & 0 & \beta & -(a + d + \mu) & 0 & 0 \\ 0 & 0 & 0 & d & -(\mu + h + a) & 0 \\ 0 & 0 & d & 0 & 0 & -(\mu + \rho) \end{bmatrix}$$

The determinant gives:

$\frac{1}{N}$ ((μ^5 + (h + k + ρ + b + α + d + β) μ^4 + ((b + h + k + β + α + q) ρ + (β + b + α + d + k) h + (α + d + b + β) k + (b + β) (q + α) μ^3 + (((β + b + α + d + k) h + (β + d + b + c – d) k + (b + β) (d + α)) ρ + ((α + d + b + β) k + (b + β) (d + α)) h + k (b + β) (d + α)) μ^2+ ((((β + d + b + α – d) k + (b + β) (d + α) h + k (b + β – d) (d + α)) ρ + k h (b + β) (d+ α)) μ + k ((b + β – d) (d + α) h – d β α) ρ) ωc (c^* + I* + T*) + (μ + ρ) (b + μ + β) (μ^2 + (α + r + k) μ + α k) (μ + h) N)

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The trace $= -\left(\left(\frac{\omega c\left(C^{\star}+I^{\star}+T^{\star}\right)}{N}+\alpha+r+\mu\right)+(k+\mu)+(a+q+\mu)\ (\beta+b+\mu)++(\mu+h)\right.$

$\left.+(\mu+\rho)\right)$ (3.70)

Since the trace $(J)<0$ and the determinant > 0, hence the Endemic Equilibrium State is locally asymptotically stable.

Conclusion

A mathematical model for control of meningitis virus disease was developed in this study.The stability analysis of the Disease Free Equilibrium State (DFE) of the model shows that it is stable, that is the population is sustainable.The analysis of the Endemic Equilibrium State (EES) shows that it is locally asymptotically stable. This means that the disease will die down.

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