

# Design Study of Cooling Fin Device

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**Abstract-** In this report, basically, it has been included the analyzing of a fin of the electric water pump motor cover temperature distribution. The effect of the convective heat transfer coefficient (h) for temperature distribution when transferring heat along the fin has been analyzed. Three different 'h' values have been taken and temperature distributions have been taken by using the MATHCAD program. It is shown graphically the temperature distribution along the length of a fin when changing the 'h' value. Importance of the convective heat transfer coefficient also has been discussed. Also variation of convective heat transfer coefficient with turbulent flow and laminar flow has been discussed.

**Key words-** Cooling fins, Turbulent flow, Convective heat transfer, Fourier's law

## I. INTRODUCTION

When the heat is transferred through a body must be frequently removed by some convection process. Basically, fins can be improved the heat transfer rate by adding the surface area. Heat can be transferred in fins by two ways. Those are conduction and convection. The conduction tends to be much smaller than the convection. Therefore, when designing the fins a little control over the materials of construction, which affects conduction. But wield considerable control over fin geometry and fin density which affects convection.

Fins improve heat transfer in two ways. One way is by creating turbulent flow through fin geometry, which reduces the thermal resistance (the inverse of the heat transfer coefficient) through the nearly stagnation film that forms when a fluid flows parallel to a solid surface. A second way is by increases the heat transfer area that comes in contact with the fluid.

Fin geometries and densities that create turbulent flow and improve performance also increase pressure drop, which is a critical requirement in most high performance applications. The optimum fin geometry and fin density combination is then a compromise of performance, pressure drop, weight and size.

In this design study, I discuss the heat transfer of water pumps. When the motor of the pump is running, it is generating heat inside the pump. That heat must be removed. Otherwise the motor of the pump can be damaged. Fins of the motor cover give the major role to remove heat in convection manner.

## II. BACKGROUND THEORY

Through the fins, heat is transferring in two ways. One is heat conduction and the other is convection. When analyzing the heat transfer through the fins, Finite Element Analysis and partial differential equation can be used. Also the Fourier's Law and the heat conduction and heat convection equations can be used.

Fourier's Law,  $q = -k(dT/dx)$

Where, q = rate of heat transfer per unit area

. x = distance

T = temperature

. k = thermal conductivity in (W/mk)

Heat conduction equation,  $dT/dt = \alpha \cdot d^2T/dx^2$

Where,  $\alpha$  = thermal diffusivity in m<sup>2</sup>/s

Heat convection equation,  $q = h \cdot A(T - T_\infty)$

Where, T = temperature on surface

$T_\infty$  = surrounding temperature

. h = convective heat transfer coefficient

## III. THEORY

Let us consider a tapered bar, which is conducting heat from one end to the other.

Assumptions

- Steady state condition.
- Uniform material properties of the bar. (independent of temperature)
- Uniformly tapering the cross section.
- Uniform convection across the surface area.
- No internal heat generation.
- One-dimensional conduction.

When heat is conducting through the bar, heat is transferred to the surrounding via heat convection. To analyse the situation, the bar can be divided into number of elements and a single element can be considered.

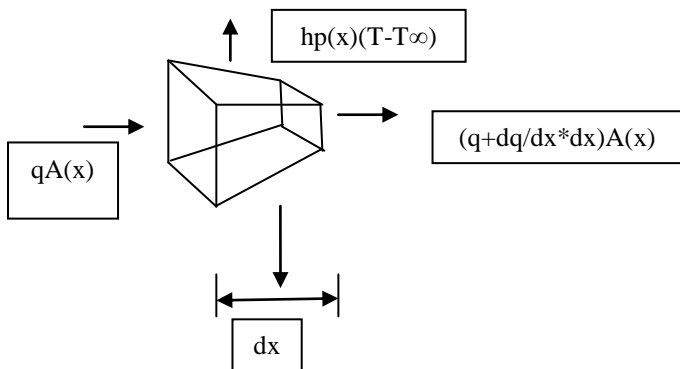


Figure 01 – Element of the fin

Then the heat balancing equation can be taken as follows.

$$qA(x) + QA(x)dx = hp(x)(T - T_{\infty}) + (q + dq/dx \cdot dx)A(x)$$

Therefore,

$$dq/dx \cdot dx \cdot A(x) + QA(x)dx = hp(x)(T - T_{\infty})$$

Also, to simplify the problem, first the heat conduction only can be analyzed along the bar. Then the weighted residual form of the heat conduction equation is given as follows.

$$\int w \{ d/dx [A(x)Kx \cdot dT/dx] + QA(x) \} dx = 0$$

Carrying out the integration of above equation, the elemental thermal conductivity matrix 'K' and flux vector 'f' are obtained as follows,

$$K = \int dNT/dx \cdot Kx \cdot d(NT)/dx \cdot A(x) dx$$

$f = \int NT QA(x) dx$  If we consider the cross sectional area  $A(x)$  as  $(N1Ai + N2Aj)$ , following equations can be taken for K and f.

Where,  $Ai$  and  $Aj$  are the cross sectional areas at two ends of  $i$  and  $j$  respectively.

$$K = Kx/L \cdot (Ai + Aj)/2 \cdot \begin{Bmatrix} 1 & -1 \\ -1 & 1 \end{Bmatrix}$$

$$f = QL/6 \cdot \begin{Bmatrix} 2Ai + Aj \\ Ai + 2Aj \end{Bmatrix}$$

Now the convective heat transfer can be considered along the bar.

Heat loss due to convection from the surface,

$$= h \cdot P(x) \cdot (T - T_{\infty})$$

Where,  $h$  = convective heat transfer coefficient

$P(x)$  = perimeter at  $x$

By considering the energy balance, the governing differential equation is as follows.

$$d/dx [A(x) \cdot Kx \cdot dT/dx] + QA(x) - hP(x)(T - T_{\infty}) = 0$$

Now consider, an element of length 'L' with nodes  $i$  and  $j$ .

If the perimeter shape function is given as follows,

$$P(x) = N1 \cdot Pi + N2 \cdot Pj$$

Also, if the Nodal temperatures are  $Ti$  and  $Tj$ , the temperature variation along the element in shape function is given as follows.

$$T(x) = N1 \cdot Ti + N2 \cdot Tj$$

By applying the Galerkin's principle the following results can be taken.

**\*results are attached in the appendix.**

#### IV. DESIGN PARAMETERS OF THE FIN OF THE MOTOR COVER OF THE ELECTRIC WATER PUMP

The fin is made of Aluminium with the thermal conductivity of 200 W/m C. The fin is rectangular cross section with thickness vary from 1cm to 0.25cm, from base to outer end as shown below.

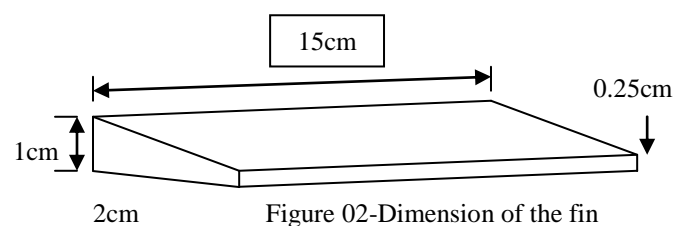


Figure 02-Dimension of the fin

$$\text{Perimeter} := \begin{pmatrix} 32 \\ 31.62 \\ 31.25 \\ 30.87 \\ 30.50 \end{pmatrix} \cdot 10^{-2}$$

- Temperature at the base = 50 C
- Ambient temperature at outside = 20 C

- Convective heat transfer coefficient = 8 W/m<sup>2</sup> C

- Internal heat generation = 4 W/m<sup>3</sup>

V. METHODOLOGY FOR ANALYZING

When the fin is analyzed, it is considered that convective heat transfer is along the 2cm length and its smaller cross section end. MATHCAD programming can be used for the analyzing.

First, the fin can be divided in to four elements of 2cm length. Then the x- coordinates of five nodes can be as (0, 0.5, 1.0, 1.5, 2.0). Corresponding nodal areas and perimeters can be taken as follows.

TABLE I. PARAMETERS AND AREAS OF NODAL

Nodal	Perimeters /cm	Nodal area /cm <sup>2</sup>
0	32.00	15.00
0.5	31.62	12.19
1.0	31.25	9.37
1.5	30.87	6.56
2.0	30.50	3.75

When applying the MATHCAD program to analyze the temperature variations in the fin,

The 'x' coordinates, nodal connectivity matrix 'NOC', the thermal conductivity of each element 'K', area of cross section at each node, and perimeter at each node must be define as follows.

$$x := 10^{-2} \cdot \begin{pmatrix} 0 \\ 0.5 \\ 1.0 \\ 1.5 \\ 2.0 \end{pmatrix} \quad K := \begin{pmatrix} 200 \\ 200 \\ 200 \\ 200 \end{pmatrix} \quad NOC := \begin{pmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 4 & 5 \end{pmatrix}$$

$$Area := 10^{-4} \cdot \begin{pmatrix} 15 \\ 12.19 \\ 9.375 \\ 6.56 \\ 3.75 \end{pmatrix}$$

Also the heat source 'Q', temperature of the surrounding medium 'T-inf', and convective heat transfer coefficient 'h' must be defined as follows.

$$Q:=0 \quad T\text{-inf}:=20 \quad h:= 8$$

Then the number of nodes 'NN' and number of elements 'NE' must be calculated as follows.

$$NN:=rows(x) \quad NE:=rows(NOC)$$

The S1 boundary condition must be defined as follows.

$$S1\text{-region}:= (1 \ 50)$$

In here 50 denotes the base temperature and 1 denotes the node number.

If the first column is 0, then that row is not processed.

By applying the above data to the MATHCAD program the following internal temperatures can be taken for the nodes from base to the end.

$$T(K\_G, F\_G) = \begin{pmatrix} 50 \\ 49.974 \\ 49.951 \\ 49.931 \\ 49.917 \end{pmatrix}$$

Then the above temperature variations can be taken in to a graph with respect to the distance from base to the end as follows.

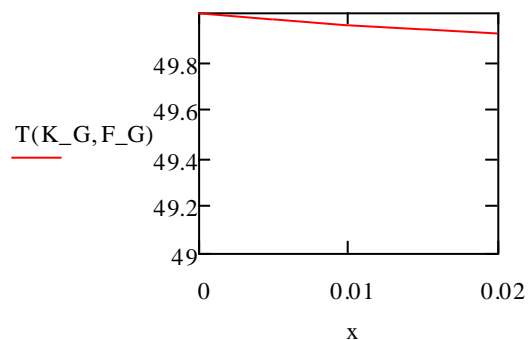


Figure 03 -Internal Temperature variation when h = 8 W/m<sup>2</sup>C

By referring the above figure 02, it can be decided the internal nodal temperature gradually decrease from base of the fin to the end of the fin. But there is no big change of the internal temperature along the length of the fin from base to the end. Number of reasons can be affected for this. That are,

- Convective heat transfer coefficient of the fin.
- Thermal conductivity of the Aluminium is very high.
- Length of the fin is not too long.
- Thickness variation of the fin.

Now the convective heat transfer coefficient can be change and the internal temperature variation of each node can be taken from the MATHCAD program as follows.

TABLE II. TEMPERATURE VARIATION WITH POWER

Nodal distance/cm	Temp /C when h=8W/m2C	Temp /C when h=15W/m2C	Temp /C when h=20W/m2C
0	50	50	50
05	49.974	49.952	49.936
10	49.951	49.908	49.877
15	49.931	49.87	49.827
20	49.917	49.845	49.794

From the above data the internal temperature variation for h=15 and h=20 also can be taken in to graphs as follows.

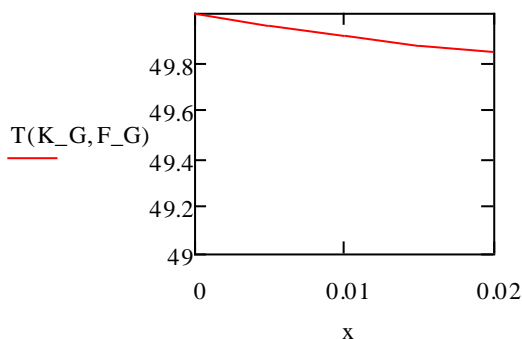


Figure 03 – Internal temperature variation when h=15W/m2C

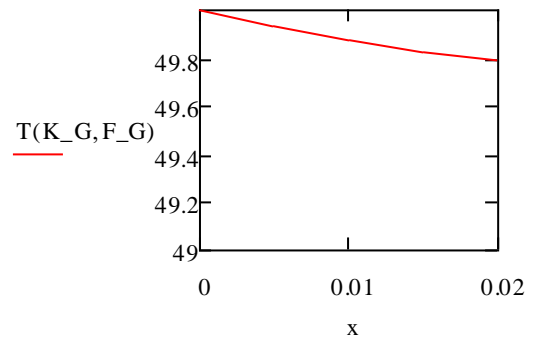


Figure 04 – Internal temperature variation when h=20W/m2

## VI. RESULTS

By examine the above figures, It can be seen that the internal temperature gradually decrease along the length of the fin from base to the end. Also it can be decided that the gradient of temperature variation is increased when the convective heat transfer coefficient is increased.

## VII. DISCUSSION



Figure 05 – Cooling fins of an electric water pump

Removing of generating heat of the motor is vital important. Otherwise the motor coil can be damaged. Increasing the temperature difference between the object and the environment, increasing the convection heat transfer coefficient, or increasing the surface area of the object increases the heat transfer.

When it is desired to increase the heat removal between a structure and a surrounding ambient fluid, it is common practice to utilized ‘extended surface’ attached to the primary surface. In such instances the extended surfaces are provided to increase artificially the surface area of the heat transmission, although the average surface temperature may be decreased by so doing. If the surface is proportioned

properly, the net result will be an increase in the heat transmission rate between the structure and the ambient fluid.

In our analyzing of the cooling fin of the pump motor is shown, that, when increasing the convective heat transfer coefficient ( $h$ ), the internal temperature will be decreased. Therefore, improving of convective heat transfer coefficient is important. Then the convective heat transfer will also improve. Let us see that, what is convection?

Convection is the term applied to the heat transfer mechanism which occurs in a fluid by the mixing of one portion of the fluid with another portion due to gross movements of the mass fluid. The actual process of energy transfer from one fluid particle or molecule to another is still one of conduction. But the energy may be transported from one point in space to another by the displacement of the fluid itself.

The fluid motion may be caused by external mechanical means (e.g. by a fan) in which case the process is called forced convection. If the fluid motion is caused by density differences existing in the fluid mass, the process is termed free convection or natural convection.

The uses of extended surfaces in applications of practical importance are numerous. Examples may be found in the cooling fins of air-cooled engines, the fin extensions to 'studs' attached to boiler tubes etc. The extended surface applications noted above are all cases in which one purposely wishes to increase the rate of heat exchange between a source and an ambient fluid. Similar extended surface configurations may occur in other instances where the exchange of heat with the ambient fluid may be a disadvantage. Such instances are encountered in the measurement of temperature.

The basic problem with which the designer is faced is; given a fin of a certain configuration and size attached to a surface of a fluid temperature, what is the rate of heat dissipated by the fin, and what is the variation in the temperature of the fin as one proceeds from the base to the tip.

When considering the Newton's law of cooling, it can be seen that the heat transfer coefficient is independent or relatively independent of the temperature difference between object and environment. This is sometimes true. But it is not guaranteed to be the case.

Also the heat transfer coefficient ' $h$ ' depends upon physical properties of the fluid and the physical situation in which convection occurs. Therefore a single usable heat transfer coefficient must be derived or found experimentally for every system analyzed.

For laminar flows, the heat transfer coefficient is rather low compared to the turbulent flows; this is due to turbulent flows having a thinner stagnation fluid film layer on heat transfer surface.

Cooling fins maintenance also important to get better convection. Debris falls on the motor, in-between cooling fins

and across air intake screen. Therefore, to get a better heat convection from cooling fins of the motor, it should be kept clean always.

Just a small amount of debris (1/2th inch) is enough to both insulate and deflect air flow away from the motor. Debris, especially pulp, acts as an insulation blanket to retain heat from the windings. Dirty and contaminated motors require frequent cleaning and maintenance.

Clogged air intakes means less air is brought into further reduce the effectiveness of the motor cooling system.

Even with a clean motor, studies have shown up to 80% of air flow is deflected away from the motor.

Hot motors consume more electricity, have degraded load capacity, require frequent maintenance on bearings and windings and operate in a low efficiency mode. But cooler motor temperature means longer bearing and winding life, fewer break downs, lower maintenance costs.

If the cooling fins remain clean and clear so air can pass through them easily. Tight fit around exterior of cooling fins provides a cooling tunnel effect to literally force the air through the cooling fins to push heat away faster and better than ever before.

## VIII. CONCLUSION

Generally heat in a body can be transferred in conduction and convection. To increase the heat transfer from a body, fins are attached to the body. Heat transfer through fins is very important. When designing fins, wield considerable control over fin geometry and fin density which affects convection.

To analyze the heat transfer through fins, the finite element analyze can be used. In this analyze, the Fourier's law and heat conduction equation can be applied. Also, for the analyzing some assumptions must be applied.

When analyzing the fin of the electric water pump motor cover, the fin has been divided in to four elements with five nodes. Then it has been analyzed by using the MATHCAD program. Results have been taken for three different values of convective heat transfer coefficient. It can be seen, when increasing the convective heat transfer coefficient, the temperature gradient of the fin increase along the length of the fin from base to the end. Convective heat transfer can be increased artificially also. The convective heat transfer is greater in turbulent flow than laminar flow.

Cooling fins must be kept clean always to get the better heat convection and the better performance.

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**APPENDIX**

$$\begin{aligned}
 \mathbf{K} &= \int_{Length_i} \frac{d\mathbf{N}^T}{dx} k_x \frac{d(\mathbf{N T})}{dx} A(x) dx \\
 &= \int_{-1}^{+1} \frac{1}{L_e} \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} k_x \frac{1}{L_e} \begin{bmatrix} -1 & 1 \end{bmatrix} \left( \frac{(1-\xi)}{2} A_i + \frac{(1+\xi)}{2} A_j \right) \frac{L_e}{2} d\xi \begin{Bmatrix} T_i \\ T_j \end{Bmatrix} \\
 \mathbf{K} &= \frac{k_x}{L_e} \left( \frac{A_i + A_j}{2} \right) \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (20.13) \\
 \mathbf{f} &= \int_{Length_i} \mathbf{N}^T Q A(x) dx = Q \int_{-1}^{+1} \begin{Bmatrix} \frac{(1-\xi)}{2} \\ \frac{(1+\xi)}{2} \end{Bmatrix} \left( \frac{(1-\xi)}{2} A_i + \frac{(1+\xi)}{2} A_j \right) \frac{L_e}{2} d\xi \\
 \mathbf{f} &= Q \frac{L_e}{6} \begin{Bmatrix} 2 A_i + A_j \\ A_i + 2 A_j \end{Bmatrix}
 \end{aligned}$$

effect of convective heat transfer along the bar is discussed now. As shown in Fig. 20.3, the heat from the surface due to convection is given by  $h P(x) (T - T_\infty)$ , where  $h$  and  $P(x)$  are the convective transfer coefficient and perimeter at  $x$  respectively. From energy balance, the governing differential equation is

$$\frac{d}{dx} \left( A(x) k_x \frac{dT}{dx} \right) + Q A(x) - h P(x) (T - T_\infty) = 0 \quad (20.14)$$

weak form of Eq. (20.14) is obtained as discussed earlier. The thermal conductivity matrix  $\mathbf{K}$  and vector  $\mathbf{f}$  are already derived in Eq. (20.13). The contribution of the convection heat transfer along element is obtained as follows. An element of length  $L$  with nodes  $i$  and  $j$  is considered. The temperature over the element is expressed using the shape functions as  $P(x) = N_1 P_i + N_2 P_j$ , where  $P_i$  and  $P_j$  are the perimeters at the ends. The linear variation of temperature along the element is also expressed using shape functions  $\mathbf{N}$  and nodal temperatures  $T_i$  and  $T_j$  as  $T(x) = N_1 T_i + N_2 T_j$ . Applying Galerkin's principle with these substitutions results in Eq. (20.15). The first part of Eq. (20.15) will be added to the element's thermal conductivity matrix  $\mathbf{K}$ . The second part of Eq. (20.15) will contribute to the element's flux vector  $\mathbf{f}$ .

$$\begin{aligned}
 \int_{Length_i} \mathbf{N}^T \left( h P(x) (T - T_\infty) \right) dx &= h \int_{Length_i} \mathbf{N}^T \mathbf{N} \begin{Bmatrix} T_i \\ T_j \end{Bmatrix} (N_1 P_i + N_2 P_j) dx \quad \mathbf{K} \\
 &\quad - h \int_{Length_i} \mathbf{N}^T (N_1 P_i + N_2 P_j) T_\infty dx \quad \mathbf{f} \\
 &= \frac{hL}{12} \begin{bmatrix} 3 P_i + P_j & P_i + P_j \\ P_i + P_j & P_i + 3 P_j \end{bmatrix} \begin{Bmatrix} T_i \\ T_j \end{Bmatrix} - \frac{h T_\infty L}{6} \begin{Bmatrix} 2 P_i + P_j \\ P_i + 2 P_j \end{Bmatrix} \quad (20.15)
 \end{aligned}$$